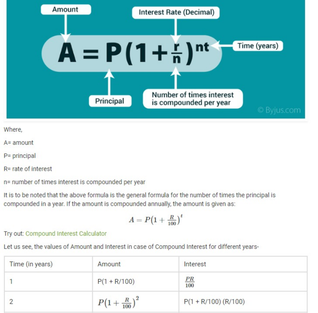
Compound Interest Formula

The compound interest formula is given below:

Compound Interest = Amount − Principal

Where the **amount** is given by:



Where,

A= amount

P= principal

R= rate of interest

n= number of times interest is compounded per year

It is to be noted that the above formula is the general formula for the number of times the principal is compounded in a year. If the amount is compounded annually, the amount is given as:

$$A = P\left(1 + \frac{R}{100}\right)^t$$

Try out: Compound Interest Calculator

Let us see, the values of Amount and Interest in case of Compound Interest for different years-

Time (in years)	Amount	Interest
1	$P(1 + R/100)$	$\frac{PR}{100}$
2	$P\left(1 + \frac{R}{100}\right)^2$	P(1 + R/100) (R/100)
3	$P\left(1 + \frac{R}{100}\right)^3$	P(1 + R/100)^2 (R/100)
4	$P\left(1 + \frac{R}{100}\right)^4$	P(1 + R/100)^3 (R/100)
n	$P\left(1 + \frac{R}{100}\right)^n$	P(1 + R/100)$^{n-1}$ (R/100)

This data will be helpful in determining the interest and amount in case of compound interest easily.

NOTE

From the data it is clear that the interest rate for the first year in compound interest is the same as that in case of simple interest, ie. $\frac{PR}{100}$

Other than the first year, the interest compound annually is always greater than that in case of simple interest.

Derivation of Compound Interest Formula

Let, Principal amount = P, Time = n years, Rate = R

Simple Interest (S.I.) for the first year:

$SI_1 = \frac{P \times R \times T}{100}$

Amount after first year = $P + SI_1 = P + \frac{P \times R \times T}{100} = P\left(1 + \frac{R}{100}\right) = P_2$

Simple Interest (S.I.) for second year:

$SI_2 = \frac{P_2 \times R \times T}{100}$

Amount after second year = $P_2 + SI_2 = P_2 + \frac{P_2 \times R \times T}{100} = P_2\left(1 + \frac{R}{100}\right) = P\left(1 + \frac{R}{100}\right)\left(1 + \frac{R}{100}\right) = P\left(1 + \frac{R}{100}\right)^2$

Similarly if we proceed further to n years, we can deduce:

$A = P\left(1 + \frac{R}{100}\right)^n$

$CI = A - P = P\left[\left(1 + \frac{R}{100}\right)^n - 1\right]$

Compound Interest when the Rate is Compounded half Yearly

Let us calculate the compound interest on a principal, P kept for 1 year at interest rate R % compounded half-yearly.

Since interest is compounded half-yearly, the principal amount will change at the end of the first 6 months. The interest for the next six months will be calculated on the amount remaining after the first six months. Simple interest at the end of first six months,

$SI_1 = \frac{P \times R \times 1}{100 \times 2}$

Amount at the end of first six months,

$A_1 = P + SI_1 = P + \frac{P \times R \times 1}{2 \times 100} = P\left(1 + \frac{R}{2 \times 100}\right) = P_2$

Simple interest for next six months, now the principal amount has changed to P_2

$SI_2 = \frac{P_2 \times R \times 1}{100 \times 2}$

Amount at the end of 1 year,

$A_2 = P_2 + SI_2 = P_2 + \frac{P_2 \times R \times 1}{2 \times 100} = P_2\left(1 + \frac{R}{2 \times 100}\right)$ = P(1 + R/2×100)(1 + R/2×100) =

$P\left(1 + \frac{R}{2 \times 100}\right)^2$

Now we have the final amount at the end of 1 year.

$A = P\left(1 + \frac{R}{2 \times 100}\right)^2$

Rearranging the above equation,

$A = P\left(1 + \frac{\frac{R}{2}}{100}\right)^{2 \times 1}$

Let $\frac{R}{2} = R'$, $2T = T'$, the above equation can be written as, [for the above case $T = 1$ year]

$A = P\left(1 + \frac{R'}{100}\right)^{T'}$

Hence, for the cases, when the rate is compounded half yearly, we divide the rate by 2 and multiply the time by 2 before using the general formula for amount in case of compound interest.

Compound Interest Quarterly Formula

Let us calculate the compound interest on a principal, P kept for 1 year at interest rate R % compounded quarterly. Since interest is compounded quarterly, the principal amount will change at the end of the first 3 months(first quarter). The interest for the next three months (second quarter) will be calculated on the amount remaining after the first 3 months. Also, interest for the third quarter will be calculated on the amount remaining after the first 6 months and for the last quarter on the remaining after the first 9 months. We can also reduce the formula of compound interest of yearly compounded for quarterly as given below:

$A = P\left(1 + \frac{R}{400}\right)^{4T}$

CI = A − P

Or

$CI = P\left(1 + \frac{R}{400}\right)^{4T} - P$

Here,

A = Amount

CI = Compound interest

R = Rate of interest per year

T = Number of years

How Compound Interest is Calculated

Let us understand the process of calculating compound interest with the help of the below example.

Example: What amount is to be repaid on a loan of Rs. 12000 for one and half years at 10% per annum compounded half yearly.

Solution:

For the given situation, we can calculate the compound interest and total amount to be repaid on a loan in two ways. In the first method, we can directly substitute the values in the formula. In the second method, compound interest can be obtained by splitting the given time bound into equal periods.

This can be well understood with the help of the table given below.

Principal for first 6 months = ₹ 12,000	**Principal for first 6 months = ₹ 12,000**
There are 3 half years in $1\frac{1}{2}$ years. Therefore, compounding has to be done 3 times. Rate of interest = half of 10% = 5% half yearly $A = P\left(1 + \frac{R}{100}\right)^n$ $= ₹\ 12000\left(1 + \frac{5}{100}\right)^3$ $= ₹\ 12000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20}$ = ₹ 13,891.50	Time = 6 months = $\frac{6}{12}$ year = $\frac{1}{2}$ year Rate = 10% $I = ₹\ \frac{12000 \times 10 \times \frac{1}{2}}{100} = ₹\ 600$ A = P + I = ₹ 12000 + ₹ 600 = ₹12600. It is principal for next 6 months. $I = ₹\ \frac{12600 \times 10 \times \frac{1}{2}}{100} = ₹\ 630$ Principal for third period = ₹ 12600 + ₹ 630 = ₹ 13,230. $I = ₹\ \frac{13230 \times 10 \times \frac{1}{2}}{100} = ₹\ 661.50$ A = P + I = ₹ 13230 + ₹ 661.50 = ₹ 13,891.50

Compound Interest vs Simple Interest

Now, let us understand the difference between the amount earned through compound interest and simple interest on a certain amount of money, say Rs. 100 in 3 years and the rate of interest is 10% p.a.

Below table shows the process of calculating interest and total amount.

		Under Simple Interest	**Under Compound Interest**
First year	Principal	₹ 100.00	₹ 100.00
	Interest at 10%	₹ 10.00	₹ 10.00
	Year-end amount	₹ 110.00	₹ 110.00
Second year	Principal	₹ 100.00	₹ 110.00
	Interest at 10%	₹ 10.00	₹ 11.00
	Year-end amount	₹(110 + 10) = ₹ 120	₹ 121.00
Third year	Principal	₹ 100.00	₹ 121.00
	Interest at 10%	₹ 10.00	₹ 12.10
	Year-end amount	₹(120 + 10) = ₹ 130	₹ 133.10

Compound Interest Examples

Let us solve various examples to understand the concepts in a better manner.

Increase or Decrease in Population

Examples 1:

A town had 10,000 residents in 2000. Its population declines at a rate of 10% per annum. What will be its total population in 2005?

Solution:

The population of the town decreases by 10% every year. Thus, it has a new population every year. So the population for the next year is calculated on the current year population. For the decrease, we have the formula A = P(1 − R/100)^n

Therefore, the population at the end of 5 years = 10000(1 − 10/100)^5

= 10000(1 − 0.1)^5 = 10000 x 0.9^5 = 5904 (Approx.)

The Growth of Bacteria

Examples 2:

The count of a certain breed of bacteria was found to increase at the rate of 2% per hour. Find the bacteria at the end of 2 hours if the count was initially 600000.

Solution:

Since the population of bacteria increases at the rate of 2% per hour, we use the formula

A = P(1 + R/100)^n

Thus, the population at the end of 2 hours = 600000(1 + 2/100)^2

= 600000(1 + 0.02)^2 = 600000(1.02)^2 = 624240

Rise or Depreciation in the Value of an Item

Examples 3:

The price of a radio is Rs 1400 and it depreciates by 8% per month. Find its value after 3 months.

Solution:

For the depreciation, we have the formula A = P(1 − R/100)^n.

Thus, the price of the radio after 3 months = 1400(1 − 8/100)^3

= 1400(1 − 0.08)^3 = 1400(0.92)^3 = Rs 1090 (Approx.)

Compound Interest Problems

Illustration 1: A sum of Rs. 10000 is borrowed by Akshit for 2 years at an interest of 10% compounded annually. Calculate the compound interest and amount he has to pay at the end of 2 years.

Solution:

Given,

Principal/ Sum = Rs. 10000, Rate = 10%, and Time = 2 years

From the table shown above it is easy to calculate the amount and interest for the second year, which is given by:

Amount$(A_2) = P\left(1 + \frac{R}{100}\right)^2$

$A_2 = 10000\left(1 + \frac{10}{100}\right)^2 = 10000\left(\frac{11}{10}\right)\left(\frac{11}{10}\right) = Rs.12100$

Compound Interest (for 2nd year) = $A_2 - P$ = 12100 − 10000 = Rs. 2100

Illustration 2: Calculate the compound interest (CI) on Rs.5000 for 2 years at 10% per annum compounded annually.

Solution:

Principal (P) = Rs.5000 , Time (T)= 2 year, Rate (R) = 10 %

We have, Amount, $A = P\left(1 + \frac{R}{100}\right)^T$

$A = 5000\left(1 + \frac{10}{100}\right)^2 = 5000\left(\frac{11}{10}\right)\left(\frac{11}{10}\right) = 50 \times 121 = Rs.\ 6050$

Interest (Second Year) = A − P = 6050 − 5000 = Rs.1050

OR

Directly we can use the formula for calculating the interest for second year, which will give us the same result.

Interest $(I_2) = P \times \frac{R}{100} = 5000 \times \frac{10}{100} = 500$

Interest $(I_2) = P \times \frac{R}{100}\left(1 + \frac{R}{100}\right) = 5000 \times \frac{10}{100}\left(1 + \frac{10}{100}\right) = 550$

Total Interest = $I_1 + I_2$ = 500 + 550 = Rs. 1050

Illustration 3: Calculate the compound interest to be paid on a loan of Rs.2000 for 3/2 years at 10% per annum compounded half-yearly?

Solution: Principal, $P = Rs.2000$, Time, $T' = 2 \times \frac{3}{2}$ years = 3 years, Rate, $R' = \frac{10}{2} = 5$, amount, A can be given as:

$A = P\left(1 + \frac{R'}{100}\right)^{6}$

$A = 2000 \times \left(1 + \frac{5}{100}\right)^3$

$= 2000 \times \left(\frac{21}{20}\right)^3 = Rs.2315.25$

$CI = A - P = Rs.2315.25 - Rs.2000 = Rs.315.25$

For detailed discussion on compound interest, download BYJU'S -The learning app. Students can also use a compound interest calculator, to solve compound interest problems in an easier way.